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A Study of the High-Energy Reaction $\bar{P}P \to \bar{P}P$

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Abstract

The high-energy behavior of the reaction $\bar{P}P \to \bar{P}P$ at $P_L = 8$ GeV/c and $P_L = 16$ GeV/c is studied in the context of O(3,1) symmetry. In view of the many exchanged particles, approximate equivalence relations in s-channel amplitudes are used which cut down the unknown parameters to a manageable set of adjustable constants. Our analysis has essentially explained the characteristic features of this reaction, which are (i) the ending of the forward peak at $|t| \simeq 0.6$ (GeV/c)^2; (ii) developing of a shoulder at larger $|t|$'s; and (iii) decreasing of the differential cross section at the shoulder for 16 GeV/c at a slower rate than 8 GeV/c.

1. *Introduction*

In this paper we have made a study of the $\bar{P}P \to \bar{P}P$ at $P_L = 8$ GeV/c and $P_L = 16$ GeV/c forward elastic scattering with a view to being able to explain the characteristic experimental features of this reaction, viz. that (i) the forward peak ends at $|t| \simeq 0.6$ GeV/c^2, (ii) a shoulder develops at larger $|t|$'s, and (iii) the cross section at the shoulder for 16 GeV/c decreases at a rate slower than 8 GeV/c.

In our study we have employed, by now, the well-known Delbourgo, Salam, and Strathdee (DSS) (1967) formalism, which is an extension of Toller's (Sciarrino & Toller, 1966) work at $t = 0$ to all values of momentum transfer. Toller essentially realizes that the elastic forward scattering amplitude admits of an $O(3,1)$ little group invariance on account of a vanishing momentum transfer four-vector.

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DSS have extended Toller's work by obtaining the expansion of the scattering amplitudes with arbitrary masses and spins.

Because of the mass shell conditions, strictly speaking, the scattering amplitudes of the inelastic processes do not have the exact $O(3,1)$ symmetry along the forward direction.

However, for small values of t in the high-energy limit, the DSS formalism has been successfully applied in the past in many cases.

The problem of forward peak is of significance in Regge-pole theory. For many reactions a simple Regge-pole model cannot explain the forward peak of the differential cross sections; a more complicated mechanism, such as conspiracy or absorptive corrections or combinations of both, is necessary to obtain agreement between experiment and theory.

The Regge-pole theory does not provide as simple an explanation as the one-pion exchange (OPE) model does with absorptive corrections, the reason being that in the Regge theory the pion is almost lost in the conspiracy between other trajectories. Raghvan and one of the authors (MS) (1969) have investigated some of the forward peak problems, and they have indicated that pions can still explain the occurrence of the forward peak with the appropriate J_0 contributions.

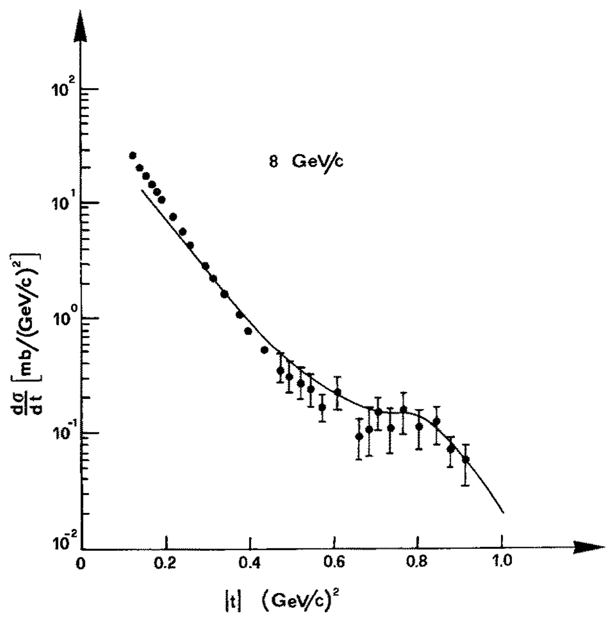

Figure 1—Differential cross section for $\bar{P}P \to \bar{P}P$ elastic scattering at P_L = 8 GeV/c. Data from Brinbaum et al. (1968).

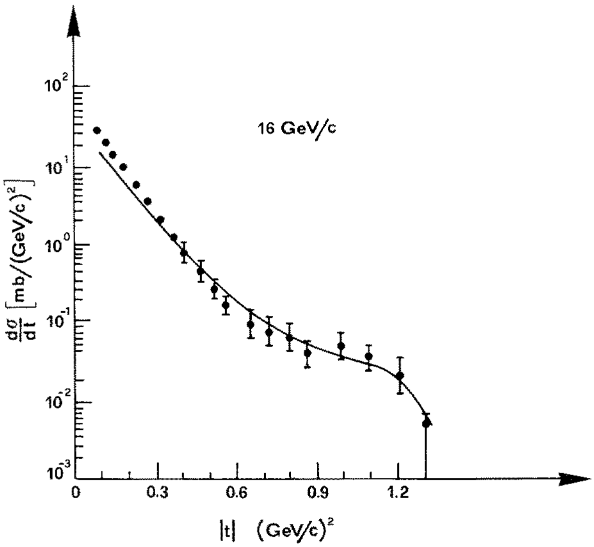

Figure 2–Differential cross section for $\bar{P}P \to \bar{P}P$ elastic scattering at P_L = 16 GeV/c. Data from Brinbaum et al. (1968).

Thus, following their suggestion, in this paper we have calculated separately the contribution of the pion and the rest of the particles with a view to finding whether the ending of the forward peak at $|t| \simeq 0.6$ (GeV/c)^2 could be explained.

Our theoretical calculations show that the pion contribution is dominant only up to $|t| \simeq 0.6$ (GeV/c)^2 and, for higher values of $|t|$, the contribution of the pion and the rest of the particles are comparable. A look at Figs. 1 and 2 will show that we have also explained the other two above-mentioned characteristic features.

2. $\bar{P}P \to \bar{P}P$ *Reactions*

For the reaction $\bar{P}P \to \bar{P}P$ the number of linearly independent scattering amplitudes is 16, if we represent them in the helicity formalism (Jacob & Wick, 1959) in the s channel as $\langle \lambda_3 \lambda_4 | T | \lambda_1 \lambda_2 \rangle$, where λ_1, λ_2, λ_3, and λ_4 are the helicities of the particles 1, 2, 3, and 4, respectively, where particles 1 and 2 are the incoming, and 3 and 4 are the outgoing ones.

After applying the relation for the conservation of parity (Cohen-Tannoudji et al., 1968)

$$\langle \lambda_3 \lambda_4 | T | \lambda_1 \lambda_2 \rangle = \eta_1 \eta_2 \eta_3 \eta_4 (-1)^{\Sigma_i (S_i + \lambda_i)} \langle -\lambda_3 - \lambda_4 | T | -\lambda_1 - \lambda_2 \rangle \quad (2.1)$$

where S_i, λ_i, and η_i denote the spin, the helicity, and the parity of the ith particle, we obtain the following eight helicity amplitudes:

$$\begin{array}{ll} \langle\tfrac{1}{2}\tfrac{1}{2}|T|\tfrac{1}{2}\,-\tfrac{1}{2}\rangle & \langle\tfrac{1}{2}\tfrac{1}{2}|T|\tfrac{1}{2}\tfrac{1}{2}\rangle \\ \langle\tfrac{1}{2}\tfrac{1}{2}|T|-\tfrac{1}{2}\tfrac{1}{2}\rangle & \langle-\tfrac{1}{2}\tfrac{1}{2}|T|-\tfrac{1}{2}\tfrac{1}{2}\rangle \\ \langle\tfrac{1}{2}\,-\tfrac{1}{2}|T|\tfrac{1}{2}\tfrac{1}{2}\rangle & \langle\tfrac{1}{2}\tfrac{1}{2}|T|-\tfrac{1}{2}\,-\tfrac{1}{2}\rangle \\ \langle-\tfrac{1}{2}\tfrac{1}{2}|T|\tfrac{1}{2}\tfrac{1}{2}\rangle & \langle\tfrac{1}{2}\,-\tfrac{1}{2}|T|-\tfrac{1}{2}\tfrac{1}{2}\rangle \end{array} \tag{2.2}$$

By an application of the formula for the time reversal invariance (Cohen-Tannoudji et al., 1968)

$$\langle\lambda_3\lambda_4|T|\lambda_1\lambda_2\rangle = (-1)^{\lambda_1-\lambda_2-\lambda_3+\lambda_4}\langle\lambda_1\lambda_2|T|\lambda_3\lambda_4\rangle \tag{2.3}$$

we find that the number of amplitudes is further reduced to the following six:

$$\begin{array}{ll} \langle\tfrac{1}{2}\tfrac{1}{2}|T|\tfrac{1}{2}\,-\tfrac{1}{2}\rangle & \langle-\tfrac{1}{2}\tfrac{1}{2}|T|-\tfrac{1}{2}\tfrac{1}{2}\rangle \\ \langle\tfrac{1}{2}\tfrac{1}{2}|T|-\tfrac{1}{2}\tfrac{1}{2}\rangle & \langle\tfrac{1}{2}\tfrac{1}{2}|T|-\tfrac{1}{2}\,-\tfrac{1}{2}\rangle \\ \langle\tfrac{1}{2}\tfrac{1}{2}|T|\tfrac{1}{2}\tfrac{1}{2}\rangle & \langle\tfrac{1}{2}\,-\tfrac{1}{2}|T|-\tfrac{1}{2}\tfrac{1}{2}\rangle. \end{array} \tag{2.4}$$

In the specific case of the nucleon-nucleon system, the total spin is conserved and can be described by the relation (Goldberger et al., 1960)

$$\langle\lambda_3\lambda_4|T|\lambda_1\lambda_2\rangle = \langle\lambda_4\lambda_3|T|\lambda_2\lambda_1| \tag{2.5}$$

A further application of Eq. (2.5) to each of the amplitudes in (2.4) yields the following five amplitudes:

$$\begin{array}{ll} \langle\tfrac{1}{2}\tfrac{1}{2}|T|\tfrac{1}{2}\,-\tfrac{1}{2}\rangle & \langle\tfrac{1}{2}\tfrac{1}{2}|T|\tfrac{1}{2}\tfrac{1}{2}\rangle \\ \langle-\tfrac{1}{2}\tfrac{1}{2}|T|-\tfrac{1}{2}\tfrac{1}{2}\rangle & \langle\tfrac{1}{2}\tfrac{1}{2}|T|-\tfrac{1}{2}\,-\tfrac{1}{2}\rangle \\ \multicolumn{2}{c}{\langle\tfrac{1}{2}\,-\tfrac{1}{2}|T|-\tfrac{1}{2}\tfrac{1}{2}\rangle.} \end{array} \tag{2.6}$$

Above, we have deliberately written our amplitudes in the s channel. This is done with a view to using the approximate equivalence relations for the s-channel helicity amplitudes developed by Cohen-Tannoudji et al. (1968). Using concepts such as the conservation of parity and G-parity at each vertex, we have further minimized the number of amplitudes, which are, as we shall see later, instrumental in cutting down enormously the number of independent parameters to be used in the investigation of the process under consideration.

The relations for the conservation of parity at the vertex (1,3) and at the vertex (2,4) formed by the incoming and the outgoing particles 1,3 and 2,4, respectively, are

$$\begin{aligned} \langle\lambda_3\lambda_4|T|\lambda_1\lambda_2\rangle &= \eta\xi\eta_1\eta_3(-1)^{S_3-S_1}(-1)^{\lambda_3-\lambda_1}\langle-\lambda_3\lambda_4|T|-\lambda_1-\lambda_2\rangle \\ \langle\lambda_3\lambda_4|T|\lambda_1\lambda_2\rangle &= \eta\zeta\eta_2\eta_4(-1)^{S_4-S_2}(-1)^{\lambda_4-\lambda_2}\langle\lambda_3\lambda_4|T|\lambda_1-\lambda_2\rangle \end{aligned} \tag{2.7}$$

where η and ζ denote the parity and signature of the particle exchanged in the t channel.

As the parity conservation at the vertex (1,3) and at the vertex (2,4) leads to the formula for the conservation of parity in the reaction, we have restricted ourselves to applying the above-given relation (2.7) for the vertex (1, 3) only.

The conservations of G parity at vertex (1,3) and vertex (2,4) are given (Cohen-Tannoudji et al., 1968) by the relations

$$\langle \lambda_3\lambda_4 | T | \lambda_1\lambda_2 \rangle \simeq g\eta(-1)^I(-1)^{\lambda_3-\lambda_1} \langle \lambda_1\lambda_4 | T | \lambda_3\lambda_2 \rangle \tag{2.8}$$

$$\langle \lambda_2\lambda_4 | T | \lambda_1\lambda_3 \rangle \simeq g\eta(-1)^I(-1)^{\lambda_4-\lambda_2} \langle \lambda_3\lambda_4 | T | \lambda_1\lambda_2 \rangle \tag{2.9}$$

Now, using parity conservation at the vertex (1,3) and conservation of G parity at both vertices, for the exchanged particles we find that only three of the five amplitudes in (2.6), i.e.,

$$\begin{aligned} &\langle \tfrac{1}{2}\tfrac{1}{2} | T | -\tfrac{1}{2} -\tfrac{1}{2} \rangle \\ &\langle \tfrac{1}{2} -\tfrac{1}{2} | T | -\tfrac{1}{2}\tfrac{1}{2} \rangle \\ &\langle \tfrac{1}{2}\tfrac{1}{2} | T | \tfrac{1}{2} -\tfrac{1}{2} \rangle \end{aligned} \tag{2.10}$$

contribute to the differential cross section. These we have given in Table 1 in the case of each particle.

Now one can use the DSS formalism and expand the helicity amplitudes in terms of the reduced $T_{S'\lambda'S\lambda}$ amplitudes as follows:

$$\langle S_3\lambda_3, S_4\lambda_4 | T | S_1\lambda_1, S_2\lambda_2 \rangle = \Sigma_{S\lambda S'\lambda'} \langle S_4\lambda_4 | S_2\lambda_2, S' - \lambda' \rangle$$
$$\times\, T_{S'\lambda'S\lambda} \langle S_3\lambda_3, S\lambda | S_1\lambda_1 \rangle \tag{2.11}$$

where S_i, λ_i are the four-momenta, spin, and helicity of the ith particle, and $\langle S_i\lambda_i | S_j\lambda_j, S\lambda \rangle$ are the Clebsch-Gordan coefficients. The $T_{S'\lambda'S\lambda}$'s are the

TABLE 1. The relevant amplitudes with dominant contribution for the exchanged particles

No.	Particle	Amplitudes
1	P	$\langle \tfrac{1}{2}\tfrac{1}{2} \| T \| \tfrac{1}{2} -\tfrac{1}{2} \rangle$ $\langle \tfrac{1}{2}\tfrac{1}{2} \| T \| -\tfrac{1}{2} -\tfrac{1}{2} \rangle$
2	P'	$\langle \tfrac{1}{2}\tfrac{1}{2} \| T \| \tfrac{1}{2} -\tfrac{1}{2} \rangle$ $\langle \tfrac{1}{2}\tfrac{1}{2} \| T \| -\tfrac{1}{2} -\tfrac{1}{2} \rangle$
3	ρ	$\langle \tfrac{1}{2}\tfrac{1}{2} \| T \| -\tfrac{1}{2} -\tfrac{1}{2} \rangle$ $\langle \tfrac{1}{2} -\tfrac{1}{2} \| T \| -\tfrac{1}{2}\tfrac{1}{2} \rangle$
4	ω	$\langle \tfrac{1}{2} -\tfrac{1}{2} \| T \| -\tfrac{1}{2}\tfrac{1}{2} \rangle$ $\langle \tfrac{1}{2}\tfrac{1}{2} \| T \| -\tfrac{1}{2} -\tfrac{1}{2} \rangle$ $\langle \tfrac{1}{2}\tfrac{1}{2} \| T \| \tfrac{1}{2} -\tfrac{1}{2} \rangle$
5	A_2	$\langle \tfrac{1}{2} -\tfrac{1}{2} \| T \| -\tfrac{1}{2}\tfrac{1}{2} \rangle$ $\langle \tfrac{1}{2}\tfrac{1}{2} \| T \| -\tfrac{1}{2} -\tfrac{1}{2} \rangle$ $\langle \tfrac{1}{2}\tfrac{1}{2} \| T \| \tfrac{1}{2} -\tfrac{1}{2} \rangle$
6	π	$\langle \tfrac{1}{2}\tfrac{1}{2} \| T \| -\tfrac{1}{2} -\tfrac{1}{2} \rangle$

reduced amplitudes. Thus, the three amplitudes in (2.10) evaluated in terms of T's read as follows:

$$\begin{aligned}\langle \tfrac{1}{2}\tfrac{1}{2}|T|-\tfrac{1}{2}-\tfrac{1}{2}\rangle &= -\tfrac{2}{3}T_{1-11-1}\\ \langle \tfrac{1}{2}-\tfrac{1}{2}|T|-\tfrac{1}{2}\tfrac{1}{2}\rangle &= \tfrac{2}{3}T_{111-1}\\ \langle \tfrac{1}{2}\tfrac{1}{2}|T|\tfrac{1}{2}-\tfrac{1}{2}\rangle &= -\sqrt{\tfrac{2}{3}}T_{1-100}-\tfrac{2}{3}T_{1-110}\end{aligned} \tag{2.12}$$

The reduced amplitudes $T_{S'\lambda'S\lambda}$ can be expressed in terms of a new set of reduced amplitudes $T^{(S')}_{J'\lambda S}$ as given by Eqs. (8) and (8′) of the DSS paper (Delbourgo et al., 1967) and are given below:

$$T_{S'\lambda'S\lambda}(s,t) = \sum_{J'} |t|^{|\Delta|/2}\langle S'\lambda'||\Delta|\Delta, J'\lambda\rangle T^{(S')}_{J'\lambda S}(s,t) \tag{2.13}$$

with

$$(\lambda \neq 0, \lambda' \neq 0),\ (\lambda = 0, \lambda' = 0),\ (\lambda \neq 0, \lambda' = 0)$$

and

$$T_{S'\lambda'S\lambda}(s,t) = \sum_{J} |t|^{|\Delta|/2} T^{S}_{S'\lambda J}\langle J\lambda'||\Delta|\Delta, S\lambda\rangle \tag{2.14}$$

with

$$\lambda = 0,\ \lambda' \neq 0$$

where

$$-\Delta \equiv (\lambda_1-\lambda_2)-(\lambda_3-\lambda_4) = (\lambda_1-\lambda_3)-(\lambda_2-\lambda_4) = \lambda - \lambda' \tag{2.15}$$

In Eqs. (2.13) and (2.14) [which are Eqs. (8) and (8′) respectively, of the DSS paper] we have adopted the notation $T^{(S')}_{J'\lambda S}$, $T^{(S)}_{S'\lambda' J}$ of Rashid and Samiullah (1967) instead of $\langle S'T'|T_\lambda|S\rangle$ and $\langle S'|T_{\lambda'}|SJ\rangle$ of the DSS paper.

Using Eqs. (2.13) and (2.14) for the process under consideration, we find the following expansions for the new set of reduced amplitudes in terms of the old ones:

$$\begin{aligned}T_{1-11-1} &= T^1_{1-11}\\ T_{111-1} &= |t|\left(\sqrt{\tfrac{2}{3}}T^1_{1-11}+\sqrt{\tfrac{1}{5}}T^1_{2-11}+\sqrt{\tfrac{1}{35}}T^1_{3-11}\right)\\ T_{1-100} &= |t|^{1/2}T^0_{1-11}\\ T_{1-110} &= \sqrt{\tfrac{1}{2}}|t|^{1/2}T^1_{1-12}\end{aligned} \tag{2.16}$$

Again following DSS, we expand these new amplitudes as follows:

$$T^{S'}_{J'\lambda S}(s,t) = \frac{1}{2\pi i}\sum_{J_0=0}^{\min(J',S)} \tfrac{1}{2}\int_{-i\infty}^{i\infty} d\sigma(J_0{}^2-\sigma^2)[d^{J_0\sigma}_{S\lambda J'}(\xi_t)+(-1)^{S+J'} \times d^{-J_0\sigma}_{S\lambda J'}(\xi t)]\ T^{S'}_{J'S}(J_0,\sigma,t) \tag{2.17}$$

where σ and J_0 are related to the two Casimir operators and the $d^{J_0\sigma}_{S\lambda J'}(\xi)$'s are the matrix elements of the principal series unitary representation of the homogeneous Lorentz group. The angle ξ_t is given in our case as

$$\cosh(\xi_t) = (s-u)/(4m_p^2 - t) \tag{2.18}$$

In the high-energy limit we write it as

$$\xi_t = 2(s-u)/(4m_p^2 - t) \tag{2.19}$$

The amplitude $T^{(S')}_{J'\lambda S}(s, t)$ given by formula (2.17) is explicitly parity invariant (Antoniou et al., 1967).

The parameter J_0 takes integral or half-integral values, while σ is pure imaginary for the principal series representations of the homogeneous Lorentz group.

Now, making the usual dynamical assumptions about the meromorphy of the amplitudes $T^{(S')}_{J'\lambda S}(s, t)$ in the complex σ plane, we may determine the asymptotic behavior of $T^{(S')}_{J'\lambda S}(s, t)$ in terms of the leading Lorentz trajectory $\sigma(t)$. Thus with the above assumptions we can recast (2.17) as follows:

$$T^{(S')}_{J'\lambda S} = \sum_{J_0=0}^{\min(J', S)} \tfrac{1}{2}(J_0{}^2 - \sigma_K{}^2)\beta^{(S')}_{J'S}[d^{J_0\sigma}_{S\lambda J'} + (-1)^{S+J'} d^{-J_0\sigma}_{S\lambda J}(\xi_t)] \tag{2.20}$$

The summation over J_0 extends over its non-negative values only, and for $J_0 = 0$ an additional factor $\frac{1}{2}$ enters on the right-hand side of (2.20). The pole is inserted at the value $\sigma = \sigma_k$ and the $\beta^{(S')}_{J'S}$ are the residues for amplitudes $J^{(S')}_{J'S}(J_0, \sigma, t)$ at $\sigma = \sigma_k$.

It has been shown by Sciarrino and Toller (1966) and later on by Iverson (1967) that at $t = 0$ a connection between the Lorentz pole and the usual Regge pole is given as

$$\alpha_n(0) = \alpha(0) - n - 1, \quad n = 0, 1, 2, \ldots \tag{2.21}$$

where $\alpha_n(0)$ is the Regge intercept of the exchanged trajectory at $t = 0$. In the preceding relation, each Lorentz pole at $\boldsymbol{\sigma} - 1$ corresponds to an infinite number of Regge daughter trajectories α_n, parallel to the parent, with their residues related to the leading one through factors which enter the group theoretic decomposition of $O(3,1)$ into $O(2,1)$ representation. In their paper, DSS have anticipated an extension of this result to arbitrary t, rewriting the relation as $\alpha_n(t) = \sigma(t) - n - 1$ with the assumption that the high-energy behavior of the amplitudes $T^{(S')}_{J'\lambda S}$ is dominated by the leading Regge trajectory. We can determine the asymptotic behavior of $T^{(S')}_{J'\lambda S}$ in terms of the Lorentz trajectory $\sigma(t)$ (Delbourgo et al., 1967) and then reexpress it in terms of the leading Regge trajectory using the formula $\sigma(t) = \alpha(t) - 1$.

Now using the relation (2.20) for each of the amplitudes in (2.16) we obtain

$$
\begin{aligned}
T^{1}_{1-11} &= -\beta^{1}_{11}\sigma^{2}d^{0\sigma}_{1-11} + \tfrac{1}{2}\tilde{\beta}^{1}_{11}(1-\sigma^{2})[d^{1\sigma}_{1-11} + d^{-1\sigma}_{\bar{1}-11}] \\
T^{1}_{2-11} &= -\tfrac{1}{2}\beta^{1}_{21}(1-\sigma^{2})[d^{1\sigma}_{1-12} - d^{-1\sigma}_{\bar{1}-11}] \\
T^{1}_{3-11} &= -2\beta^{1}_{31}\sigma^{2}d^{0\sigma}_{1-13} + \tfrac{1}{2}\tilde{\beta}^{1}_{31}(1-\sigma^{2})[d^{1\sigma}_{1-13} + d^{-1\sigma}_{\bar{1}-13}) \\
T^{0}_{1-11} &= -\beta^{0}_{11}\sigma^{2}d^{0\sigma}_{1-11} + \tfrac{1}{2}\beta^{0}_{11}(1-\sigma^{2})[d^{1\sigma}_{1-11} + d^{-1\sigma}_{\bar{1}-11}] \\
T^{1}_{1-12} &= \tfrac{1}{2}\beta^{1}_{12}(1-\sigma^{2})[d^{1\sigma}_{2-11} - d^{-1\sigma}_{\bar{2}-11}]
\end{aligned}
\tag{2.22}
$$

Explicitly evaluating the asymptotic form of the d functions in the high-energy limit, we obtain the expression for the dominant contributions of the helicity amplitudes given in Table 2.

In order to further minimize the arbitrary number of parameters, we have analyzed the experimental data using the amplitudes $\langle \frac{1}{2}\frac{1}{2}|T|-\frac{1}{2}-\frac{1}{2}\rangle$ and $\langle \frac{1}{2}\frac{1}{2}|T|\frac{1}{2}-\frac{1}{2}\rangle$ only, with $J_0 = 0$ only.

For the differences in theoretical calculations and the experimental data, one can use the other amplitudes if desirable.

For P, P', and ω we have used the trajectories given by Flores-Maldonado (1967), viz.

$$\alpha_P = 1.3t, \quad \alpha_{P'} = 0.7 + 1.21t, \quad \alpha_\omega = 0.5 + 0.7t$$

which have been previously studied in connection with the analysis of NN and

TABLE 2. The contribution of $J_0 = 0$ and $J_0 = 1$ for the asymptotic helicity amplitudes[a]

Helicity amplitude	$J_0 = 0$	$J_0 = 1$
$\langle \frac{1}{2}\frac{1}{2}\|T\|-\frac{1}{2}-\frac{1}{2}\rangle$	$2\beta^{1}_{11}\dfrac{\sigma}{\sigma+1}S'^{\,\sigma-2}$	$2\tilde{\beta}^{1}_{11}(\sigma-1)S'^{\,\sigma-1}$
$\langle \frac{1}{2}-\frac{1}{2}\|T\|-\frac{1}{2}\frac{1}{2}\rangle$	$-\|t\|\left(\dfrac{6}{\sqrt{15}}\beta^{1}_{11} + \dfrac{2\sqrt{2}}{\sqrt{5}}\beta^{1}_{31}\right)$	$\|t\|\left(\dfrac{3}{\sqrt{15}}\tilde{\beta}^{1}_{11} + \dfrac{1}{\sqrt{15}}\tilde{\beta}^{1}_{31}\right.$
	$\times\left(\dfrac{\sigma}{\sigma+1}\right)S'^{\sigma-2}$	$\left.+\dfrac{2(\sigma-2)}{\sigma(\sigma+2)}\tilde{\beta}^{1}_{21}\right)(1-\sigma)S'^{\sigma-1}$
$\langle \frac{1}{2}\frac{1}{2}\|T\|\frac{1}{2}-\frac{1}{2}\rangle$	$-\|t\|^{1/2}\sqrt{6}\,\beta^{0}_{11}\dfrac{\sigma}{\sigma+1}S'^{\,\sigma-2}$	$\|t\|^{1/2}\left(-\sqrt{6}\,\tilde{\beta}^{1}_{11}\right.$
		$\left.-\dfrac{\sqrt{15}}{6}\tilde{\beta}^{1}_{12}\dfrac{\sigma-2}{\sigma+2}\right)$
		$\times(\sigma-1)S'^{\,\sigma-1}$

[a] Read $S' = \dfrac{2(s-u)}{4m_p^2 - t}$

TABLE 3. Theoretical values of the differential cross section for the momentum of the incident antiproton at P_L = 8 GeV/c in mb/(GeV/c)^2

$-t$	$\frac{d\sigma}{dt}(P, P', \omega, \rho, A_2)$	$\frac{d\sigma}{dt}(\pi)$	$\frac{d\sigma}{dt}(P, P', \omega, \rho, A_2, \& \pi)$
0.05	0.3754	4.1289	7.0594
0.10	0.3115	3.0320	5.2821
0.15	0.2601	2.1940	3.9649
0.20	0.2187	1.5820	2.9771
0.30	0.1574	0.8108	1.3256
0.40	0.1159	0.4041	0.9529
0.50	0.0871	0.1928	0.5389
0.60	0.0665	0.0856	0.3031
0.70	0.0517	0.0335	0.1685
0.80	0.0406	0.0042	0.1652
0.90	0.0323	0.0015	0.0478
1.00	0.0260	0.0001	0.0298

TABLE 4. Theoretical values of the differential cross section for the momentum of the incident antiproton at P_L = 16 GeV/c in mb/(GeV/c)^2

$-t$	$\frac{d\sigma}{dt}(P, P', \rho, \omega, A_2)$	$\frac{d\sigma}{dt}(\pi)$	$\frac{d\sigma}{dt}(P, P', \omega, \rho, A_2, \& \pi)$
0.05	0.4430	9.7506	14.2585
0.10	0.3594	6.5881	10.0250
0.15	0.2942	4.4740	7.0629
0.20	0.2430	3.0277	4.9861
0.30	0.1696	1.3668	2.4996
0.40	0.1218	0.6000	1.2625
0.50	0.0895	0.2521	0.6419
0.60	0.0670	0.0986	0.2679
0.70	0.0511	0.0340	0.1684
0.80	0.0394	0.0091	0.0865
0.90	0.0308	0.0012	0.0442
1.00	0.2430	0.0001	0.0268
1.20	0.0155	0.0000	0.0155
1.30	0.0117	0.0000	0.0117

$N\bar{N}$ elastic scattering data. In the literature, several trajectories have been proposed for A_2 and ρ (Thews, 1967; Brower & Dash, 1968; Arnold, 1967). In this work we have used the trajectory given by Krammer (1967) for A_2, $\alpha_{A_2} = 0.4 + 0.9t$, and the popular parametrization for ρ, $\alpha_\rho = 0.58 + 0.90t$.

For the pion, we have used $\alpha_\pi = -0.02 + t$ (Raghvan & Samiullah, 1969). In our work we have treated the residues as adjustable parameters independent of t. Our theoretical values of the differential cross section for the momentum of the incident antiproton at $P_L = 8$ GeV/c and at $P_L = 16$ GeV/c are given in Tables 3 and 4 respectively.

In Figs. 1 and 2 we have plotted the theoretical values of the differential cross section with respect to $|t|$ at $P_L = 8$ GeV/c and $P_L = 16$ GeV/c, respectively.

3. *Results and Discussion*

We have analyzed the antiproton-proton elastic scattering data for the lab momentum of the incident antiproton at $P_L = 8$ GeV/c and $P_L = 16$ GeV/c of the BNL-Carnegie Mellon group (Brinberg et al., 1968). We find that the essential features of the experimental data are correctly reproduced.

A look at Tables 3 and 4 shows that the forward scattering peak exists mostly due to the pion contribution, and its ending at $|t| \simeq 0.6$ (GeV/c)^2 is explained by the fact that for both $P_L = 8$ GeV/c and $P_L = 16$ GeV/c the pion contribution, although it decreases continuously, is dominant only up to $|t| \simeq 0.6$ (GeV/c)^2, and later on it becomes comparable to the sum of the contributions of all the other exchanged particles, explaining the occurrence of a shoulder. At higher values of t it tends to zero, and thus there is a drop in the value of the differential cross section at these values.

If we imagine a line drawn through the 8 GeV/c data and reproduce it on a 16 GeV/c graph, it would reveal that the forward peak continues expanding with an increasing energy, and also that the cross section decreases at the shoulder with increasing energy.

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